

Mass distributions for induced fission of different Hg isotopes

A.V.Andreev, G.G.Adamian, N.V.Antonenko

Joint Institute for Nuclear Research, 141980 Dubna, Russia

With the improved scission-point model the mass distributions are calculated for induced fission of different Hg isotopes with the masses 180–196. The drastic change in the shape of the mass distribution from asymmetric to symmetric is revealed with increasing mass number of the fissioning Hg isotope, and the reactions are proposed to verify this prediction experimentally. The asymmetric mass distribution of fission fragments observed in the recent experiment on the fission of ^{180}Hg is explained. The calculated mass distribution and mean total kinetic energy of fission fragments are in a good agreement with the available experimental data.

PACS numbers: 24.75.+i, 25.85.-w, 24.10.Pa

Key words: Binary fission; Mass distribution; Scission point model; Dinuclear system model

Since the discovery of nuclear fission this phenomenon is intensively investigated. The mass distributions in the low-energy fission of the nuclei of actinide region were explored in details. The asymmetric shape of the mass distribution is well known in the spontaneous, neutron induced, and β -delayed fission of most actinide isotopes. Such asymmetric shape was theoretically explained by taking into account the shell structure of the fragments [1–3]. At the last decades new experimental techniques were developed to be able to investigate the low-energy fission of lighter isotopes. The studies of Coulomb-excited fission of radioactive beams [4] revealed the predominance of symmetric fission in the light thorium to astatine region. In the fission of stable targets with masses 185–210 induced by bombardment with protons and ^3He [5] the mass distribution was also found to be symmetric in most cases. However, for several nuclei with the masses about 200 the mass distribution looks symmetric but with a small dip at the top. Based on the most of experimental data, one could conclude that the asymmetric shape of mass distribution in low-energy fission changes to symmetric with decreasing mass number of the fissioning nucleus. It was unexpected that in the recent experiment [6] on β -delayed fission of ^{180}Tl (the fissioning nucleus is ^{180}Hg) the shape of the mass distribution was found to be clearly asymmetric. The explanation of this outstanding

result is a challenge for nuclear theory and a good test for the existing models of nuclear fission.

As shown in Refs. [7, 8], the observable characteristics of fission process are formed near the prescission configurations of fissioning nucleus. Indeed, with the modified scission-point model [9] one can describe the experimental data on fission of actinides: mass, charge, and kinetic energy distributions, neutron multiplicity distributions. A new explanation of a bimodality effect in fission of heavy actinides and fine structure of mass-energy distribution in fission of ^{236}U have been proposed. Our model have been also extended to the description of ternary fission [10]. The advantage of our model is that it allows us to describe various experimental data with the fixed set of parameters and assumptions. The wide range of described fission observables and effects demonstrates the predictive power of the model. In the present work we apply our model to the fission of lighter nuclei for describing the new experimental data on asymmetric fission of ^{180}Hg .

Here, we give a short description of the model, the details can be found in Refs. [9, 10]. The fissioning nucleus at the scission point is modeled by the two nearly touching coaxial spheroids — fragments of a dinuclear system with the masses A_L , A_H and charges Z_L , Z_H of the light (L) and heavy (H) fragments, respectively. $A = A_L + A_H$ and $Z = Z_L + Z_H$ are the mass and charge numbers of a fissioning nucleus, respectively. Taking into account the volume conservation, the shape of the system is defined by the mass and charge numbers of the fragments, deformation parameters of the fragments β_i , and interfragment distance R . The deformation parameter of each fragment is the ratio of the major and minor semi-axes of the spheroid $\beta_i = c_i/a_i$. Here and further, $i = L, H$ denotes the light and heavy fragments of the dinuclear system. The case $\beta = 1$ corresponds to the spherical shape of the fragment. For small values of β , the following equality is valid: $\beta \approx \beta_2 + 1$, where β_2 is the parameter of quadrupole deformation in the multiple expansion of the fragment shape.

The potential energy of the system is the sum of the liquid drop energies U_i^{LD} of each fragment, the shell correction terms δU_i^{sh} , the energy of interaction of the fragments $V^C + V^N$, and the rotational energy V^{rot} . The shell corrections are calculated with the Strutinsky method and two-center shell model [11], the damping of the shell corrections with excitation is introduced in our model. The interaction energy consists of the Coulomb interaction of two uniformly charged spheroids and nuclear interaction in the form of a double folding of

nuclear densities and density-dependent Skyrme-type nucleon-nucleon forces [12].

$$\begin{aligned}
U(\{A_i, Z_i, \beta_i\}, R) = & U_L^{LD}(A_L, Z_L, \beta_L) + U_H^{LD}(A_H, Z_H, \beta_H) \\
& + \delta U_L^{sh}(A_L, Z_L, \beta_L) + \delta U_H^{sh}(A_H, Z_H, \beta_H) \\
& + V^C(\{A_i, Z_i, \beta_i\}, R) + V^N(\{A_i, Z_i, \beta_i\}, R) \\
& + V^{rot}(\{A_i, Z_i, \beta_i\}, R, l).
\end{aligned} \tag{1}$$

All these terms, including the shell correction terms, depend on deformations of the fragments. For given deformations of the fragments the nuclear interaction potential has a potential minimum (pocket) as a function of the interfragment distance R . For calculation of the potential energy we take the value of interfragment distance corresponding to this minimum. Depending on the masses of the fragments and their deformations the calculated distance between the tips of the spheroids is 0.5–1 fm.

The thermodynamical equilibrium is postulated at the scission point. The excitation energy of the nuclear system at scission is calculated as a difference between the potential energy $U_{g.s.}$ of the fissioning nucleus in the ground state and the potential energy U of dinuclear system at the scission point plus the initial excitation energy of the fissioning nucleus: $E^* = U_{g.s.} - U + E_{CN}^*$. The temperature is calculated as $T = \sqrt{E^*/a}$, where $a = A/12$ is the level density parameter in the Fermi-gas model. The yield of a particular scission configuration with given mass and charge numbers and deformation parameters of the fragments is proportional to the exponential Boltzmann-factor:

$$Y(\{A_i, Z_i, \beta_i\}) \sim \exp \left\{ -\frac{U(\{A_i, Z_i, \beta_i\})}{T} \right\}. \tag{2}$$

For given mass and charge split, the potential energy of the dinuclear system at the scission point is a function of deformations of the fragments, and the potential energy surface (PES) $U_{\{A_i, Z_i\}}(\beta_L, \beta_H)$ can be drawn. Due to the Coulomb interaction between the fragments, the deformation parameters corresponding to the minimum of PES are larger than in the ground states of nuclei-fragments, that indicates that the fragments at the scission point are significantly deformed. To obtain the relative mass distribution as a function of the mass number of one of the fragments in fission of a compound nucleus with mass and charge numbers A and Z , one should integrate the expression (2) over Z_L , β_L , and β_H , and take

into account that $A_H = A - A_L$ and $Z_H = Z - Z_L$:

$$Y(A_L) = \frac{\int \exp \left\{ -\frac{U(\{A_i, Z_i, \beta_i\})}{T} \right\} dZ_L d\beta_L d\beta_H}{\int \exp \left\{ -\frac{U(\{A_i, Z_i, \beta_i\})}{T} \right\} dA_L dZ_L d\beta_L d\beta_H}. \quad (3)$$

This distribution is normalized to unity.

We made calculations of mass distributions for five isotopes of Hg with the mass numbers 180, 184, 188, 192, and 196. To reduce the computation time, the calculations were restricted only to even-even fragments since their yield is maximal and defines the shape of the mass distribution. The inclusion of the odd-even and odd-odd fragments can only smooth out a little the distribution but can not appreciably change its shape.

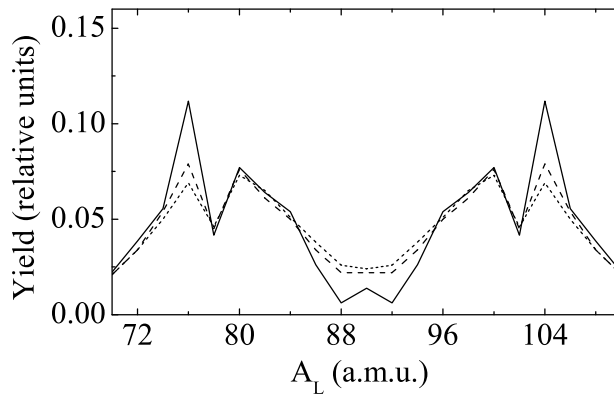


FIG. 1: Calculated mass distributions of fission fragments for β -delayed fission of ^{180}Tl (solid line), and for induced fission of ^{180}Hg with the impact energies of 10 MeV (dashed line) and 30 MeV (dotted line) above the Coulomb barrier.

For β -delayed fission of ^{180}Tl [6], the excitation energy E_{CN}^* of the fissioning nucleus ^{180}Hg does not exceed 10.44 MeV. The result of calculations is presented in Fig. 1. We obtained sharply asymmetric mass distribution with the maximum of the yield for $^{76}\text{Se}+^{104}\text{Pd}$, while the experiment gives the maximum of the yield for $^{80}\text{Kr}+^{100}\text{Ru}$. Within the experimental uncertainty of 2 units in extraction of fragment charge number this result seems to be in a rather good quantitative agreement with the experimental data, while qualitative agreement is excellent. Note that for $^{80}\text{Kr}+^{100}\text{Ru}$ we got also the maximum.

Figure 1 demonstrates the influence of the excitation energy of the fissioning nucleus ^{180}Hg on the shape of the mass distribution. The excitation energy reduces the shell effects

and smooths out the shape of the mass distribution. However, the influence of the excitation energy is not so dramatic. The mass distribution have a pronounced asymmetric shape even at the excitation energy $E_{CN}^*=64.2$ MeV.

If one excludes the shell correction terms from Eq. (1) the PES will have a minimum at the deformations of the fragments of about $\beta_i=1.6$ (see Fig. 2), which are larger than the ground state deformations of the corresponding nuclei because of the polarization effect. Hence, the shell effects at these deformations play crucial role for calculation of the yield, while the shell effects at small deformations play almost no role, since the rest part of the energy is high there.

In fission of ^{180}Hg the symmetric scission configuration is $^{90}\text{Zr}+^{90}\text{Zr}$. The shell correction for ^{90}Zr has a negative value $\delta U^{sh} \approx -2$ MeV near $\beta=0$, at larger deformations it becomes positive, at $\beta=1.6$ it is equal to $\delta U^{sh} \approx 1$ MeV, then grows further and at $\beta=1.85$ reaches $\delta U^{sh} \approx 4$ MeV. This increases the energy of the scission configuration at β around 1.6 and reduces the yield of corresponding fragments. On the contrary, the shell corrections for non-magic nuclei in the scission configurations Kr+Ru and Se+Pd are usually positive at small deformations ($\delta U_L^{sh} \approx 2.5$ MeV, $\delta U_H^{sh} \approx 1.5$ MeV) and have zero or slightly negative values in the region around $\beta=1.6$ which reduces the energy of the scission configurations and increases their contribution. The width of the minimum of PES plays also a significant role. Due to the integration in Eq. (3), the wide minimum results in the larger yield of corresponding fragments. Because of the strong shell effects, the minimum is narrow for $^{90}\text{Zr}+^{90}\text{Zr}$, while for $^{76}\text{Se}+^{104}\text{Pd}$, where the shell effects are weaker, it is wide (see Fig. 2). This also leads to a relative decrease of the yield of symmetric mass split. Hence, in the fission of ^{180}Hg we obtain the asymmetric mass distribution with a minimum at Zr+Zr, maximum at Se+Pd, and rather large yield of Kr+Ru.

For all considered isotopes ^{180}Hg , ^{184}Hg , ^{188}Hg , ^{192}Hg , and ^{196}Hg we proposed the reactions and performed the calculations of induced fission with impact energies 10 MeV and 30 MeV above the Coulomb barriers V_b : $^{36}\text{Ar}+^{144}\text{Sm} \rightarrow ^{180}\text{Hg}$ ($V_b=126.2$ MeV); $^{40}\text{Ar}+^{144}\text{Sm} \rightarrow ^{184}\text{Hg}$ ($V_b=124.55$ MeV); $^{40}\text{Ar}+^{148}\text{Sm} \rightarrow ^{188}\text{Hg}$ ($V_b=123.9$ MeV); $^{32}\text{S}+^{160}\text{Gd} \rightarrow ^{192}\text{Hg}$ ($V_b=114.4$ MeV); $^{36}\text{S}+^{160}\text{Gd} \rightarrow ^{196}\text{Hg}$ ($V_b=112.8$ MeV). Figure 3 shows a drastic change in the shape of the mass distribution with increasing mass number of the fissioning nucleus. While the mass distribution is sharply asymmetric for ^{180}Hg , for ^{184}Hg the mass distribution is rather flat, for ^{188}Hg the mass distribution is symmetric but

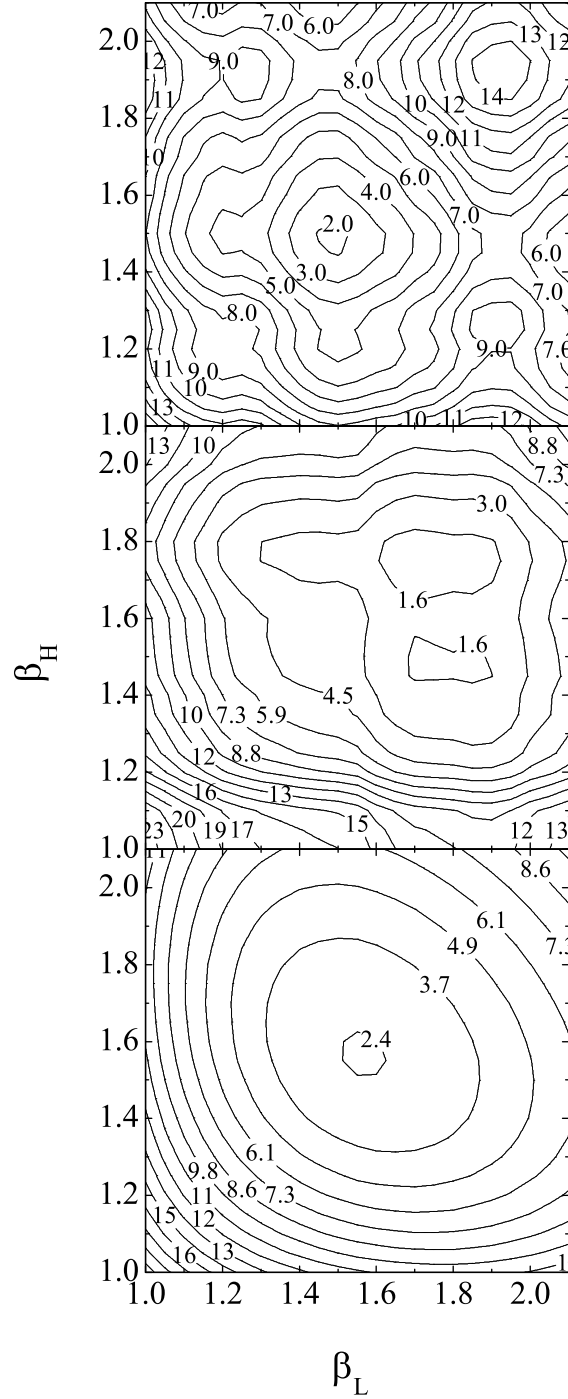


FIG. 2: Calculated potential energy at the scission point as a function of deformations of the fragments in the binary systems $^{90}\text{Zr}+^{90}\text{Zr}$ (upper part) and $^{76}\text{Se}+^{104}\text{Pd}$ (middle part). The energy is given in MeV relative to the energy of the fissioning nucleus ^{180}Hg . For the binary system $^{90}\text{Zr}+^{90}\text{Zr}$, the potential energy calculated without shell correction terms in Eq. (1) is shown (bottom part).

very wide, and for ^{192}Hg and ^{196}Hg the mass distribution have a sharply symmetric shape.

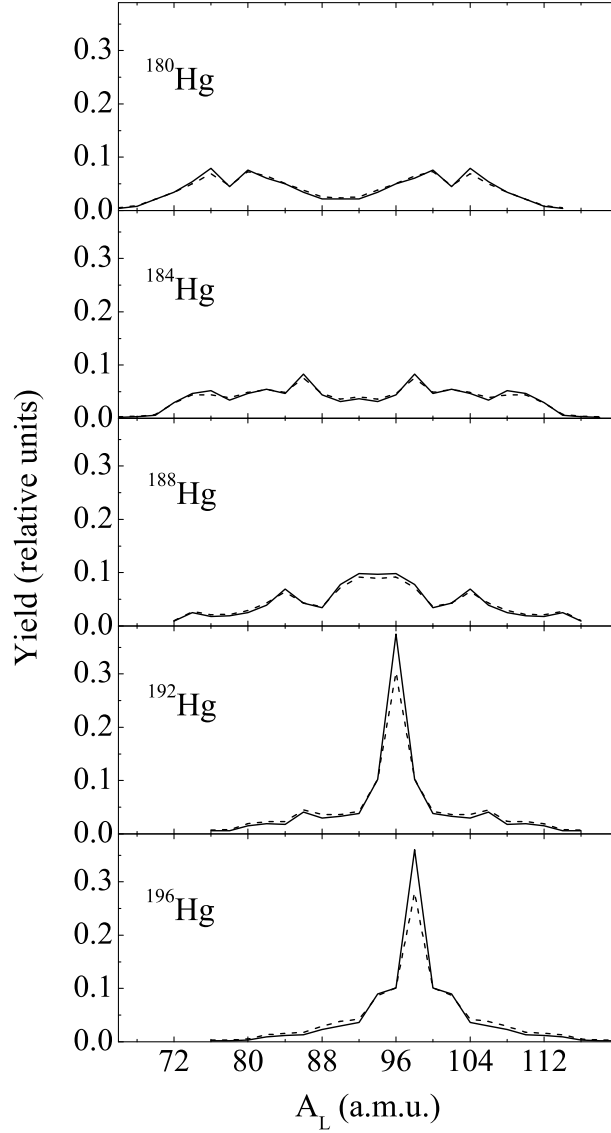


FIG. 3: Calculated mass distributions of fission fragments for induced fission of ^{180}Hg , ^{184}Hg , ^{188}Hg , ^{192}Hg , and ^{196}Hg with the impact energies of 10 MeV (solid lines, $E_{CN}^*=44.2$ MeV, 43.9 MeV, 49.7 MeV, 62.4 MeV, 56.0 MeV for $^{180,184,188,192,196}\text{Hg}$, respectively) and 30 MeV (dashed lines, $E_{CN}^*=64.2$ MeV, 63.9 MeV, 69.7 MeV, 82.4 MeV, 76.0 MeV for $^{180,184,188,192,196}\text{Hg}$, respectively) above the Coulomb barrier.

As known [1–3], the mass distribution have always a symmetric shape if the shell effects are not taken into account. In the present calculations the main role in the formation of the mass distribution plays the shell effects in nuclei with a magic neutron number $N = 50$

because the symmetric split of ^{180}Hg consists of two magic nuclei ^{90}Zr while asymmetric splits of ^{180}Hg consist of non-magic nuclei. This marks out the isotope ^{180}Hg and neighboring nuclei among others. In the heavier isotopes of Hg only one fission fragment can have the neutron number $N = 50$, and hence, the influence of the shell effects is not so strong in these nuclei comparing to ^{180}Hg .

With increasing mass of the fissioning nucleus, the fragments of symmetric scission configurations deviates from the magic ^{90}Zr , and the role of strong shell effects at symmetric splits decreases. Thereby, in the heavy isotopes of Hg the shape of the mass distribution is generally defined by the liquid-drop part of the energy, and we obtain a symmetric mass distribution.

The scission-point model is also suitable for describing the total kinetic energy (TKE) of the fission fragments. We calculate the TKE supposing that all interaction energy at the scission point transforms after fission into the kinetic energy of the fission fragments. Therefore, the value of the TKE strongly depends on the deformations of the fragments at the scission point. The smaller the deformations of the fragments, the larger the Coulomb repulsion, the larger the TKE. The mean value of the total kinetic energy for particular binary splitting is calculated by averaging over deformations of the fragments on the PES:

$$\langle TKE \rangle(\{A_i, Z_i\}) = \frac{\int [V^C(\{A_i, Z_i, \beta_i\}) + V^N(\{A_i, Z_i, \beta_i\})] \exp \left\{ -\frac{U(\{A_i, Z_i, \beta_i\})}{T} \right\} d\beta_L d\beta_H}{\int \exp \left\{ -\frac{U(\{A_i, Z_i, \beta_i\})}{T} \right\} d\beta_L d\beta_H}. \quad (4)$$

Thus, the value of $\langle TKE \rangle$ is generally defined by the position of the minimum of PES. In the cases of two magic nuclei $^{90}\text{Zr} + ^{90}\text{Zr}$ and neighboring dinuclear systems the shell corrections are strong and negative at small deformations and grows with increasing deformations. Hence, the minimum of the potential energy is shifted from the liquid drop minimum $\beta=1.6$ to smaller deformations $\beta=1.5$ that leads to relative increase of $\langle TKE \rangle$. In the scission configurations consisting of non-magic nuclei the minimum of the potential energy correspond to liquid-drop minimum with the deformations around $\beta=1.6$ – 1.65 . Figure 4 demonstrates the calculated dependence of $\langle TKE \rangle$ on the mass number of the light fission fragment for the induced fission with the impact energy 10 MeV above the Coulomb barrier. The curve rises fast for ^{180}Hg due to approaching to the compact symmetric scission configuration $^{90}\text{Zr} + ^{90}\text{Zr}$, while for ^{196}Hg the curve is almost horizontal since the symmetric configuration $^{98}\text{Zr} + ^{98}\text{Zr}$ consist of non-magic nuclei. It can be interesting to measure such dependences experimentally and to compare with the results of our theoretical predictions.

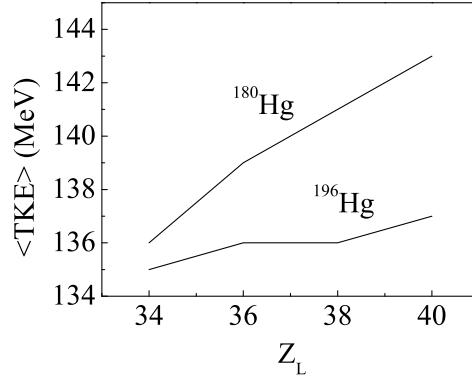


FIG. 4: The mean TKE of fission fragments for the induced fission of ^{180}Hg and ^{196}Hg as functions of the mass number of light fragment.

The average value of TKE of the fission fragments can be found by averaging over all binary systems:

$$\overline{TKE} = \int \langle TKE \rangle(\{A_i, Z_i\}) Y(\{A_i, Z_i\}) dA_L dZ_L, \quad (5)$$

where

$$Y(A_i, Z_i) = \frac{\int \exp \left\{ -\frac{U(\{A_i, Z_i, \beta_i\})}{T} \right\} d\beta_L d\beta_H}{\int \exp \left\{ -\frac{U(\{A_i, Z_i, \beta_i\})}{T} \right\} dA_L dZ_L d\beta_L d\beta_H}. \quad (6)$$

For β -delayed fission of ^{180}Tl , we obtained the average TKE of 136 MeV, that is in a good agreement with the experimental result [6].

The results of our calculations confirm the importance of the shell structure in the fission process. The account of different fragment deformations at the scission point is necessary for the correct description of the mass distributions and kinetic energy of the fission fragments. Our model gives a good description of the recent experiment, where the asymmetric mass distribution in fission of ^{180}Hg was observed. This unexpected effect required a theoretical explanation and the present work provides it. We made a prediction of the change in the shape of the mass distribution from asymmetric to symmetric with increasing mass number of the fissioning Hg isotope, and proposed the reactions to verify this prediction experimentally.

We thank Dr. A. N. Andreyev for fruitful discussions. This work was supported by DFG (Bonn) and RFBR (Moscow).

-
- [1] F. Gönnenwein, in *Nuclear Fission Process*, edited by C. Wagemans (CRC Press, Boca Raton, FL, 1991).
 - [2] H. L. Hall and D. C. Hoffman, J. Radiol. Nucl. Chem. **142**, 53 (1990).
 - [3] Yu. Ts. Oganessian, J. Phys. G **34**, R165 (2007).
 - [4] K.-H. Schmidt *et al.*, Nucl. Phys. A **693**, 169 (2001); A **665**, 221 (2000).
 - [5] M. G. Itkis *et al.*, Sov. J. Nucl. Phys. **52**, 601 (1990); **53**, 757 (1991).
 - [6] A. N. Andreyev *et al.*, Phys. Rev. Lett. **105**, 252502 (2010).
 - [7] B. D. Wilkins, E. P. Steinberg, R. R. Chasman, Phys. Rev. C **14**, 1832 (1976).
 - [8] J. Randrup, P. Möller, and A. J. Sierk, Phys Rev. C **84**, 034613 (2011).
 - [9] A. V. Andreev *et al.*, Eur. Phys. J. A **22**, 51 (2004); **26**, 327 (2005).
 - [10] A. V. Andreev *et al.*, Eur. Phys. J. A **30**, 579 (2006).
 - [11] J. Maruhn and W. Greiner, Z. Physik **251**, 431 (1972).
 - [12] G. G. Adamian *et al.*, Int. J. Mod. Phys. E **5**, 191 (1996).